IU Task D Note-015
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Determination of the Minimal Number of Partial Waves Required to Describe the $\pi^+\pi^-\pi^-$ Dataset *

Scott Teige
Department of Physics
Indiana University, Bloomington, IN 47405

Abstract

A method for determining the significance of various partial waves is discussed. The method is applied to a specific partial wave analysis to reduce the number of partial waves (and therefore fit parameteres) used to a minimum.

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1 Introduction

A Partial Wave Analysis (PWA) is and expansion of a complex amplitude that describes a particular reaction. Considered here is the reaction

$$\pi^- p \to \pi^+ \pi^- \pi^- p. \tag{1}$$

In principle, there are infinitely many basis functions (partial waves) in the expansion. In practice, the expansion is truncated at some finite number of terms. The choice of terms to include could be considered arbitrary and a source of systematic bias in the analysis. To avoid this an analytical method is used here to demonstrate certain terms in an expansion are insignificant.

A large set of partial waves was chosen as a starting point for this analysis. Individual partial waves were removed systematically and the change in the $\ln(\mathcal{L})$ was calculated. Table 2 gives a list of the partial waves used in this initial analysis.

Since the computer time required for PWA to complete depends quadratically on the number of partial waves included in the fit, considerable savings of time can be achieved by removing partial waves in batches. That is, the large partial wave set is used only initially to evaluate the significance of a certain class of partial waves. Once the insignificant partial waves in this class are determined they are removed from the expansion and the reduced set of partial waves used to determine the significance of other partial waves.

Since each partial wave using the $\pi\pi$ S-wave isobar contributes 4 parameters to the fit, these were examined first. The seven partial waves containing this isobar were removed one at a time and the resulting change in the $\ln(\mathcal{L})$ function evaluated. A change of four units (one for each parameter) is expected simply because the number of degrees of freedom is increased when parameters are removed.

Figure 1 shows the result of removing an insignificant partial wave. The top two panels in this figure show the individual intensities associated with each component of the $\pi\pi$ S-wave isobar partial wave removed for this comparison. Note that these individual intensities are usually separated from zero by approximately one error bar. The bottom panel in this figure shows the change in $\ln(\mathcal{L})$ associated with removing these partial waves. The horizontal line is a a change of 4 units, the level expected from purely statistical consideration. The change everywhere is close to this expectation with the exception of a few bins at the low edge of the mass range considered.

Figure 2 shows the effect of removing a significant partial wave from the expansion. The intensities of the components of isobar are large (compared to their errors) and exhibit structure. The change in the $\ln(\mathcal{L})$ function is large compared to four over most of the mass range considered here. It is interesting to note that this particular partial wave is significant at low mass (below 2 GeV) and insignificant at higher effective mass.

Figures 3 and 4 show two other partial waves found to be insignificant. Figures 5 through 7 show the $\pi\pi$ S-wave isobar partial waves determined to be significant at this stage of the analysis. Table 2 shows the partial waves that were considered and that survived this stage of the analysis.

The process of testing and removing partial waves was repeated twice more. The partial waves using the $f_2(1275)$ isobar (see table 3) were examined first followed by those using the ρ (see table 4).

2 Conclusion

A total of 16 partial waves were removed by $f_2(1275)$the procedure discussed above. The number of parameters used in the fit (neglecting reductions for reference waves) was reduced from 110 to 72. The resulting set of partial waves has been shown to be significant in reducing $-\ln(\mathcal{L})$ or are included for reasons of physical interest. It should be noted that all $J^{pc} = 1^{-+}$ are, at best, of marginal significance.

$J^{PC}M^{\epsilon}L\,isobar$ (Negative Refl.)	$J^{PC}M^{\epsilon}L\,isobar$ (Positive Refl.)
$1^{-+}0^{-}P\,\rho\pi$	$0^{-+}0^{+}S\,(\pi\pi$ S-wave$)\ \pi$
$1^{-+}1^{-}P\,\rho\pi$	$0^{-+}0^{+}P\,\rho\pi$
$1^{++}1^{-}S\,\rho\pi$	$0^{-+}0^{+}D\,f_2(1275)\ \pi$
$1^{++}1^{-}P\,(\pi\pi$ S-wave$)\ \pi$	$1^{-+}1^{+}P\,\rho\pi$
$1^{++}1^{-}P\,f_2(1275)\ \pi$	$1^{++}0^{+}S\,\rho\pi$
$1^{++}1^{-}D\,\rho\pi$	$1^{++}0^{+}P\,(\pi\pi$ S-wave$)\ \pi$
$2^{-+}1^{-}S\,f_2(1275)\ \pi$	$1^{++}0^{+}P\,f_2(1275)\ \pi$
$2^{-+}1^{-}P\,\rho\pi$	$1^{++}0^{+}D\,\rho\pi$
$2^{-+}1^{-}D\,(\pi\pi$ S-wave$)\ \pi$	$1^{++}1^{+}S\,\rho\pi$
$2^{-+}1^{-}D\,f_2(1275)\ \pi$	$1^{++}1^{+}P\,(\pi\pi$ S-wave$)\ \pi$
$2^{-+}1^{-}F\,\rho\pi$	$1^{++}1^{+}P\,f_2(1275)\ \pi$
$2^{++}0^{-}D\,\rho\pi$	$1^{++}1^{+}D\,\rho\pi$
$2^{++}1^{-}D\,\rho\pi$	$2^{-+}0^{+}S\,f_2(1275)\ \pi$
	$2^{-+}0^{+}P\,\rho\pi$
	$2^{-+}0^{+}P\,\rho_3\pi$
	$2^{-+}0^{+}D\,(\pi\pi$ S-wave$)\ \pi$
	$2^{-+}0^{+}D\,f_2(1275)\ \pi$
	$2^{-+}0^{+}F\,\rho\pi$
	$2^{-+}1^{+}S\,f_2(1275)\ \pi$
	$2^{-+}1^{+}P\,\rho\pi$
	$2^{-+}1^{+}P\,\rho_3\pi$
	$2^{-+}1^{+}D\,(\pi\pi$ S-wave$)\ \pi$
	$2^{-+}1^{+}D\,f_2(1275)\ \pi$
	$2^{-+}1^{+}F\,\rho\pi$
	$2^{++}1^{+}D\,\rho\pi$
	$3^{++}0^{+}S\,\rho_3\pi$
	$3^{++}0^{+}P\,f_2(1275)\ \pi$
	$3^{++}0^{+}D\,\rho\pi$
	$4^{-+}0^{+}P\,\rho_3\pi$
	$4^{-+}0^{+}D\,f_2(1275)\ \pi$
	$4^{-+}0^{+}F\,\rho\pi$
	$4^{++}1^{+}D\,\rho_3\pi$
	$4^{++}1^{+}F\,f_2(1275)\ \pi$
	$4^{++}1^{+}G\,\rho\pi$
background	

Table 1: The initial partial wave set

$J^{PC}M^{\epsilon}L\,isobar$ (Negative Refl.)		$J^{PC}M^{\epsilon}L\,isobar$ (Positive Refl.)	
$[1^{++}1^{-}P\,(\pi\pi$ S-wave$)\pi]$	(3)	$0^{-+}0^{+}S\,(\pi\pi$ S-wave$)\ \pi$	(2)
$[2^{-+}1^{-}D\,(\pi\pi$ S-wave$)\pi]$	(1)	$1^{++}0^{+}P\,(\pi\pi$ S-wave$)\ \pi$	(5)
		$[1^{++}1^{+}P\,(\pi\pi$ S-wave$)\ \pi]$	(4)
		$2^{-+}0^{+}D\,(\pi\pi$ S-wave$)\ \pi$	(6)
		$2^{-+}1^{+}D\,(\pi\pi$ S-wave$)\ \pi$	(7)

Table 2: The S-wave isobars considered in the first reduction of the wave set. Those partial waves enclosed in square brackets were removed from thewave set. The number in parentheses refers to the figure showing the intensity and the change in $\ln(\mathcal{L})$

$J^{PC}M^{\epsilon}L\,isobar$ (Negative Refl.)		$J^{PC}M^{\epsilon}L\,isobar$ (Positive Refl.)	
$[1^{++}1^{-}P\,f_2(1275)\ \pi]$	(8)	$[0^{-+}0^{+}D\,f_2(1275)\ \pi]$	(13)
$[2^{-+}1^{-}S\,f_2(1275)\ \pi]$	(9)	$[1^{++}0^{+}P\,f_2(1275)\ \pi]$	(11)
$[2^{-+}1^{-}D\,f_2(1275)\ \pi]$	(10)	$1^{++}1^{+}P\,f_2(1275)\ \pi$	(14)
		$2^{-+}1^{+}S\,f_2(1275)\ \pi$	(15)
		$2^{-+}1^{+}D\,f_2(1275)\ \pi$	(16)
		$3^{++}0^{+}P\,f_2(1275)\ \pi$	(17)
		$[4^{-+}0^{+}D\,f_2(1275)\ \pi]$	(12)
		$4^{++}1^{+}F\,f_2(1275)\ \pi$	(18)

Table 3: The $f_2(1275)$ isobars considered in the second reduction of the wave set. Those partial waves enclosed in square brackets were removed from the wave set. The number in parentheses refers to the figure showing the intensity and the change in $\ln(\mathcal{L})$

$J^{PC}M^{\epsilon}L\,isobar$ (Negative Refl.)		$J^{PC}M^{\epsilon}L\,isobar$ (Positive Refl.)	
$1^{-+}0^{-}P\,\rho\pi$	(26)	$0^{-+}0^{+}P\,\rho\pi$	(29)
$1^{-+}1^{-}P\,\rho\pi$	(27)	$1^{-+}1^{+}P\,\rho\pi$	(30)
$[1^{++}1^{-}S\,\rho\pi]$	(20)	$[1^{++}0^{+}D\,\rho\pi]$	(24)
$[1^{++}1^{-}D\,\rho\pi]$	(21)	$1^{++}1^{+}S\,\rho\pi$	(31)
$[2^{-+}1^{-}P\,\rho\pi]$	(22)	$[1^{++}1^{+}D\,\rho\pi]$	(25)
$[2^{-+}1^{-}F\,\rho\pi]$	(23)	$2^{-+}0^{+}P\,\rho\pi$	(32)
$2^{++}0^{-}D\,\rho\pi$	(28)	$2^{-+}0^{+}F\,\rho\pi$	(33)
$[2^{++}1^{-}D\,\rho\pi]$	(19)	$2^{-+}1^{+}P\,\rho\pi$	(34)
		$2^{-+}1^{+}F\,\rho\pi$	(35)
		$3^{++}0^{+}D\,\rho\pi$	(36)
		$4^{-+}0^{+}F\,\rho\pi$	(37)
		$4^{++}1^{+}G\,\rho\pi$	(38)

Table 4: The ρ isobars considered in the third reduction of the wave set. Those partial waves enclosed in square brackets were removed from the wave set. The number in parentheses refers to the figure showing the intensity and the change in $\ln(\mathcal{L})$

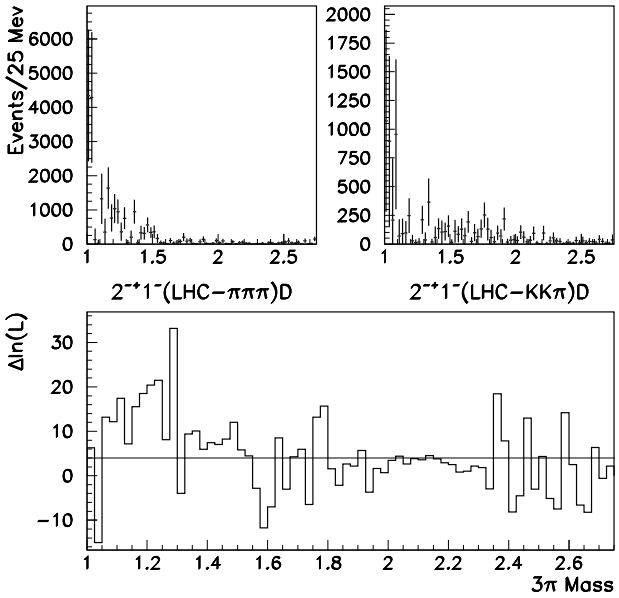

Figure 1: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves removed from the fit.

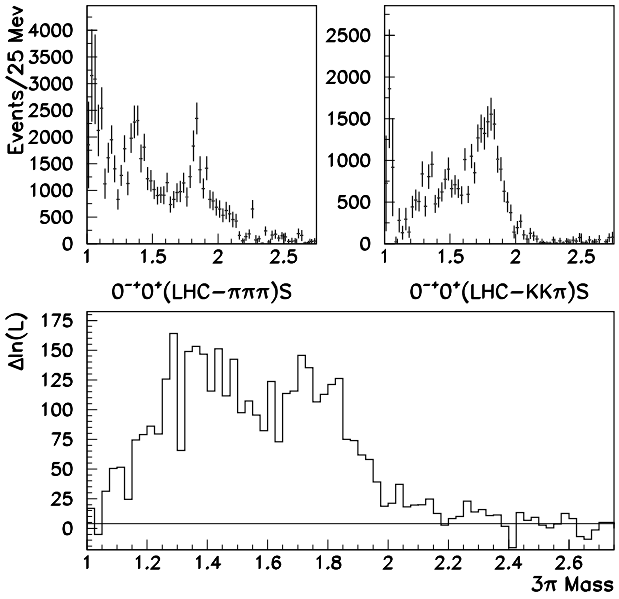

Figure 2: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves determined to be significant.

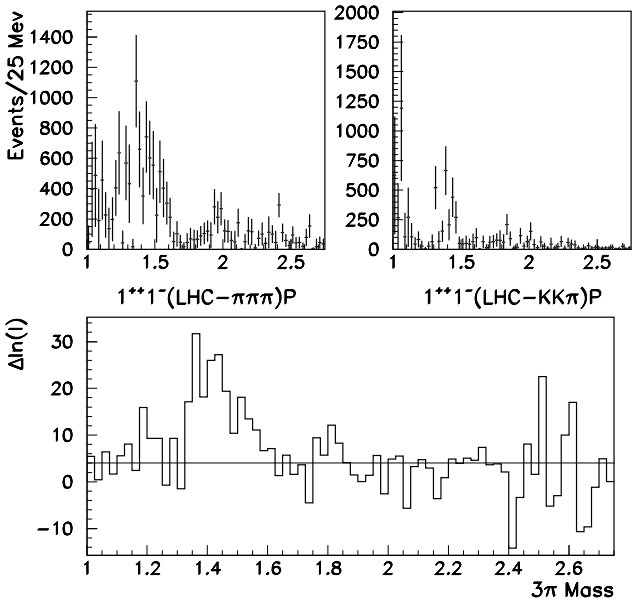

Figure 3: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves removed from the fit.

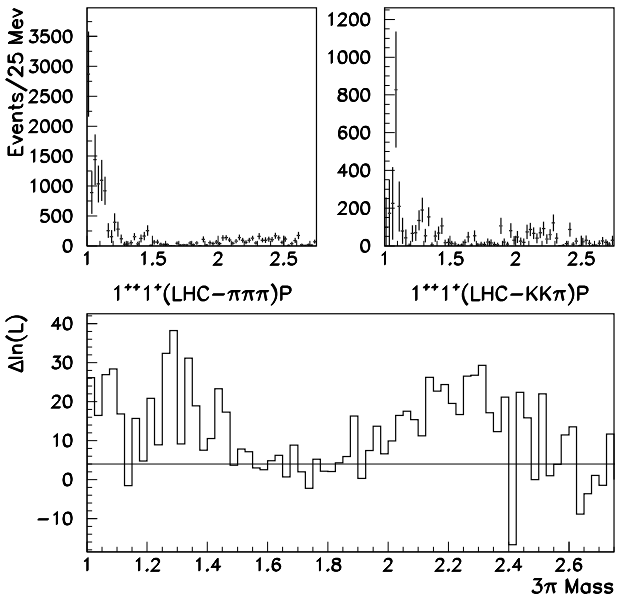

Figure 4: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves removed from the fit.

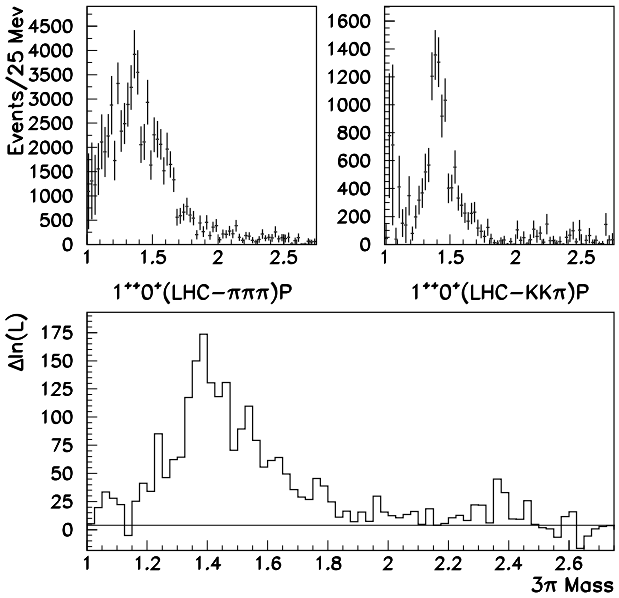

Figure 5: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves determined to be significant.

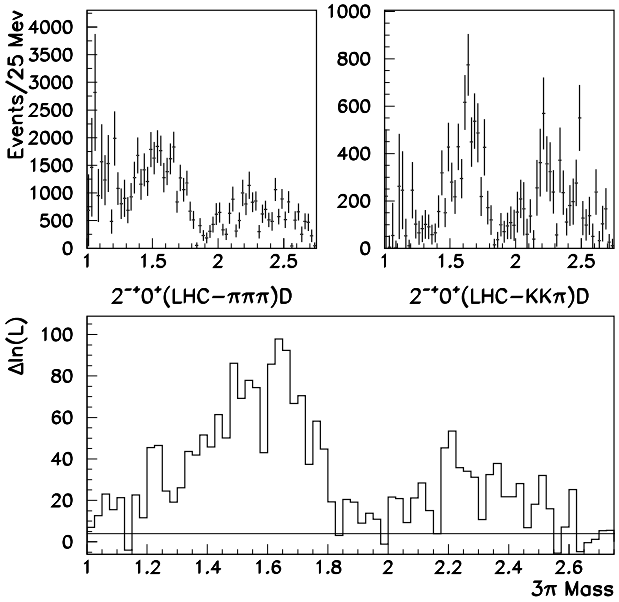

Figure 6: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves determined to be significant.

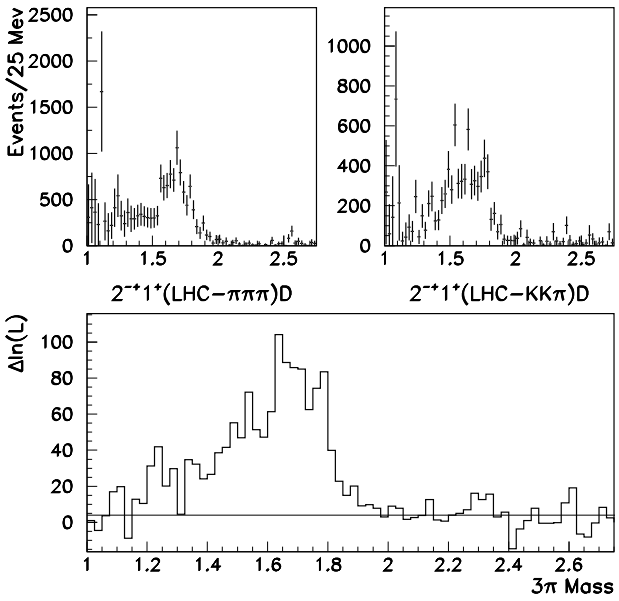

Figure 7: The Intensity (top row) and change in likelihood (bottom panel) for a pair of partial waves determined to be significant.

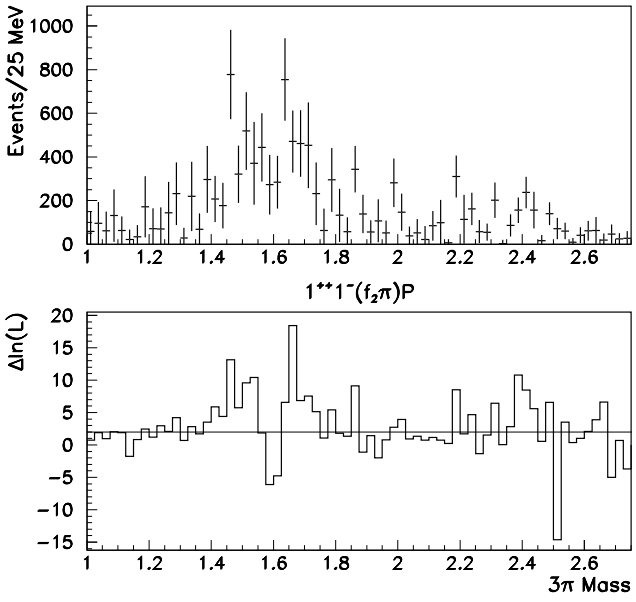

Figure 8: An insignificant partial wave

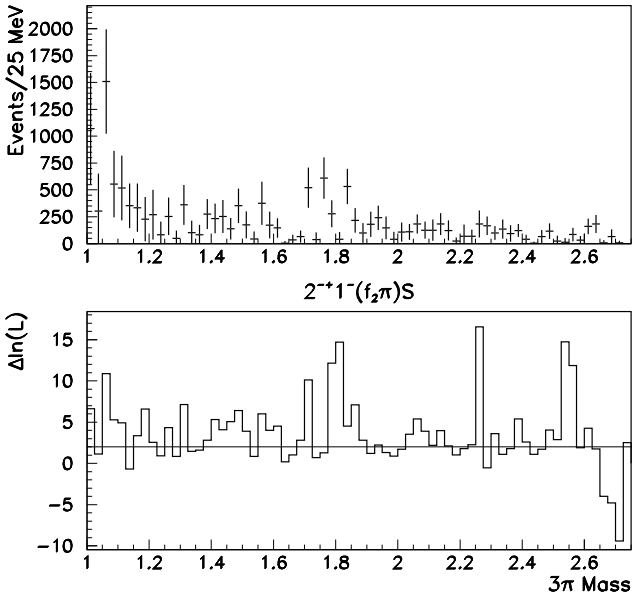

Figure 9: An insignificant partial wave

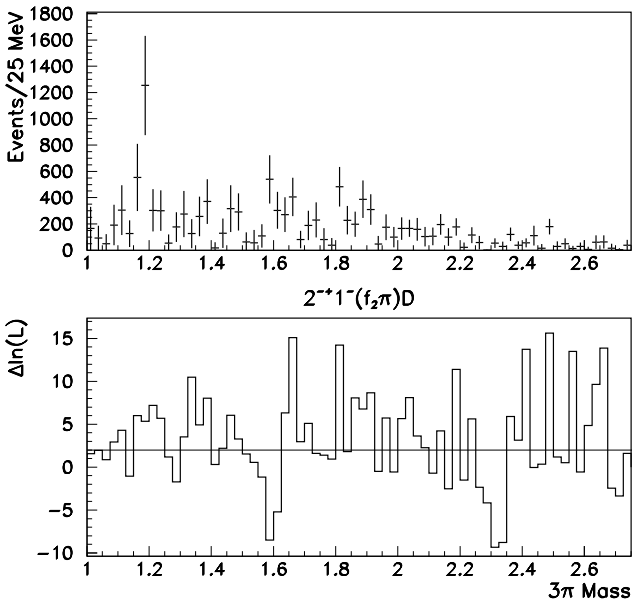

Figure 10: An insignificant partial wave

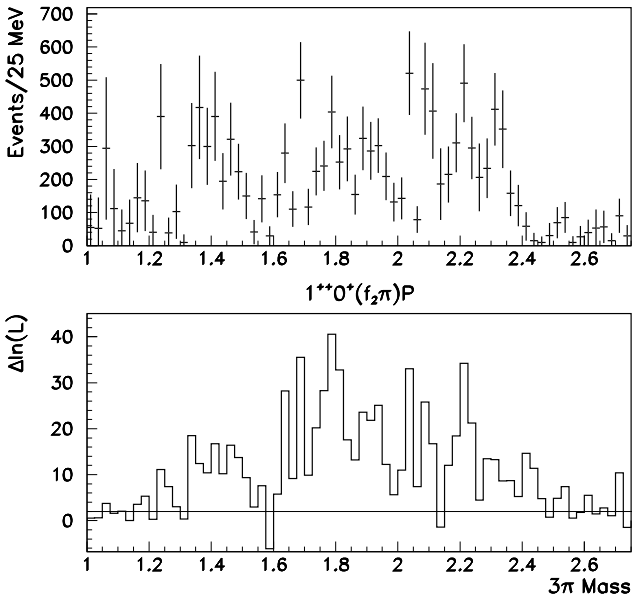

Figure 11: An insignificant partial wave

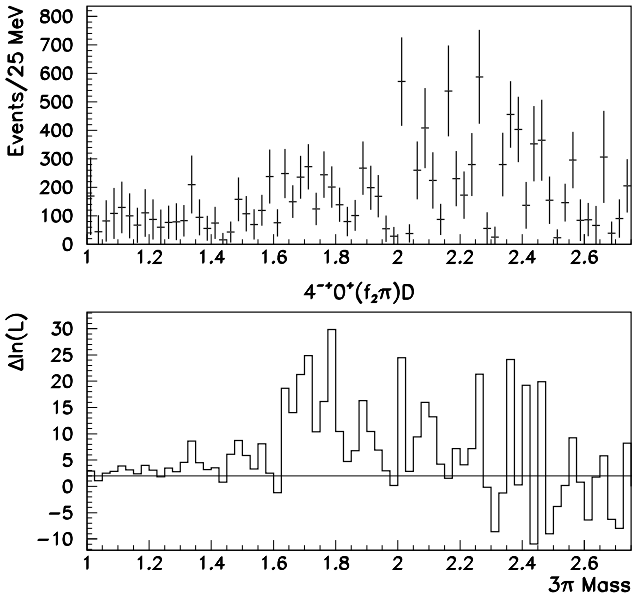

Figure 12: An insignificant partial wave

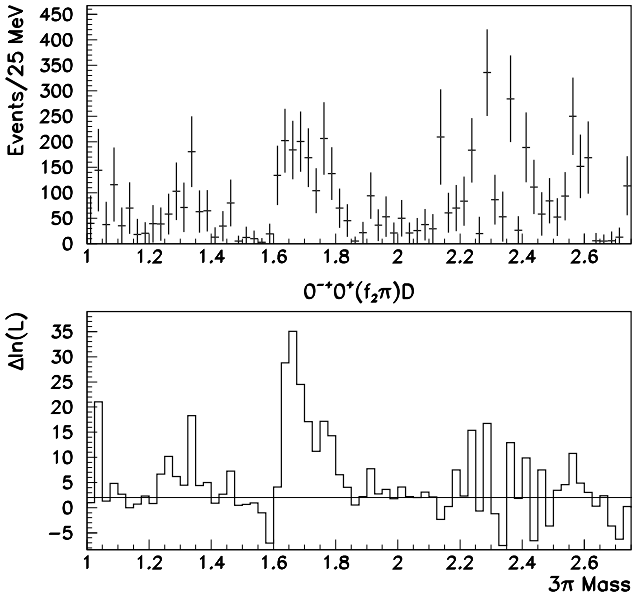

Figure 13: An insignificant partial wave

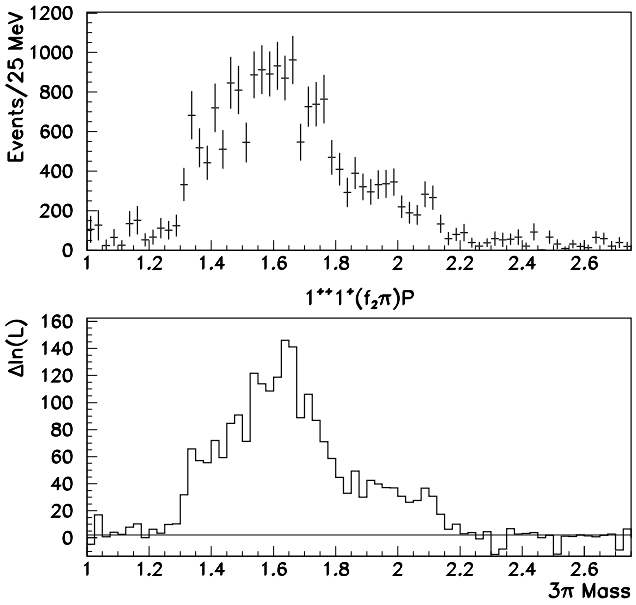

Figure 14: A significant partial wave

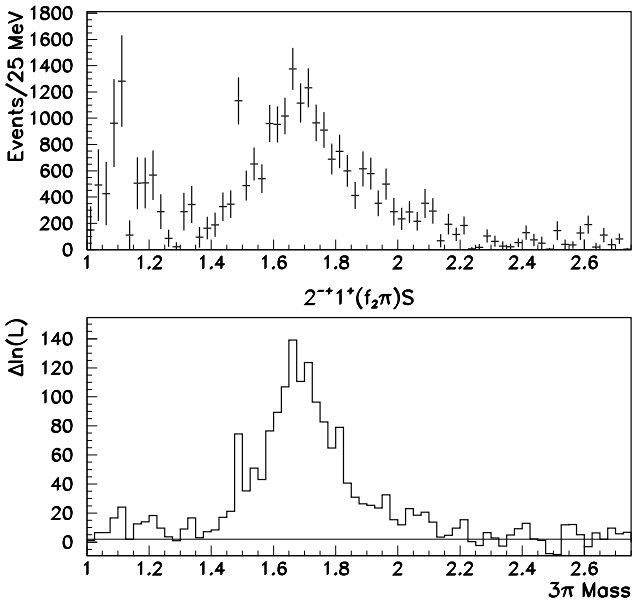

Figure 15: A significant partial wave

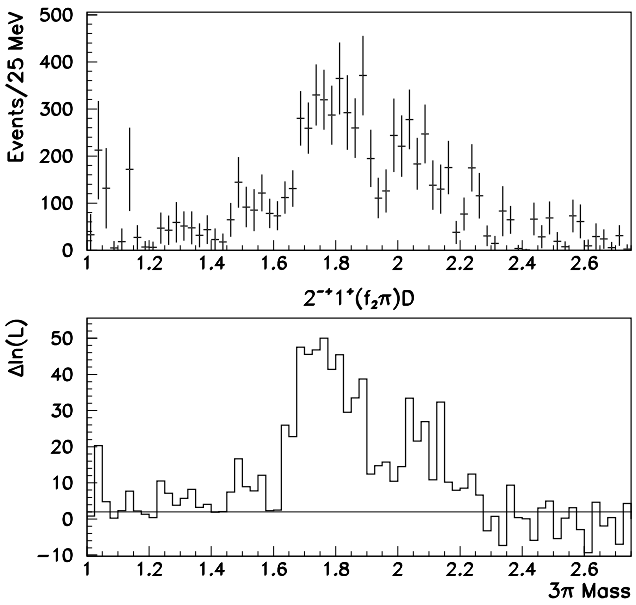

Figure 16: A significant partial wave

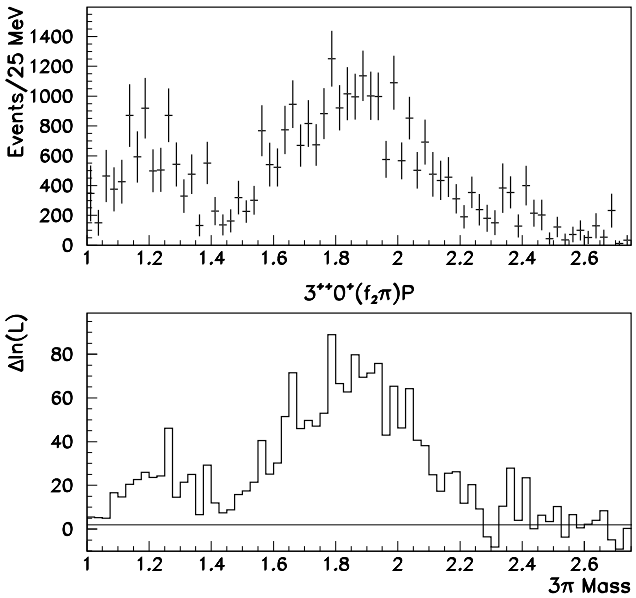

Figure 17: A significant partial wave

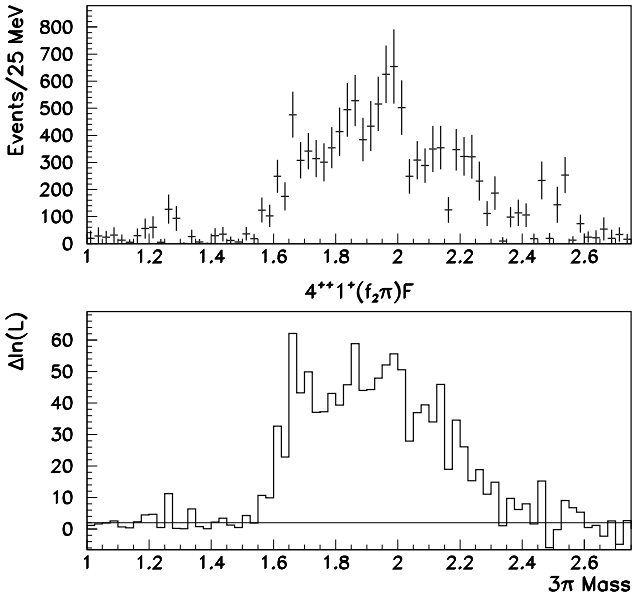

Figure 18: A significant partial wave

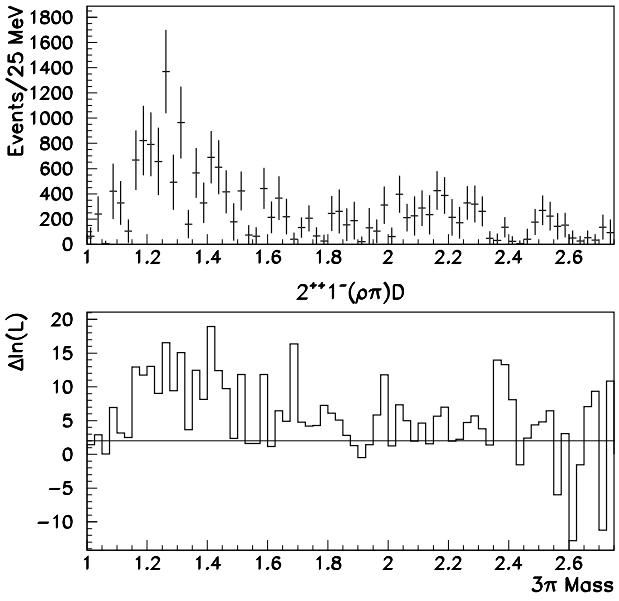

Figure 19: An insignificant partial wave

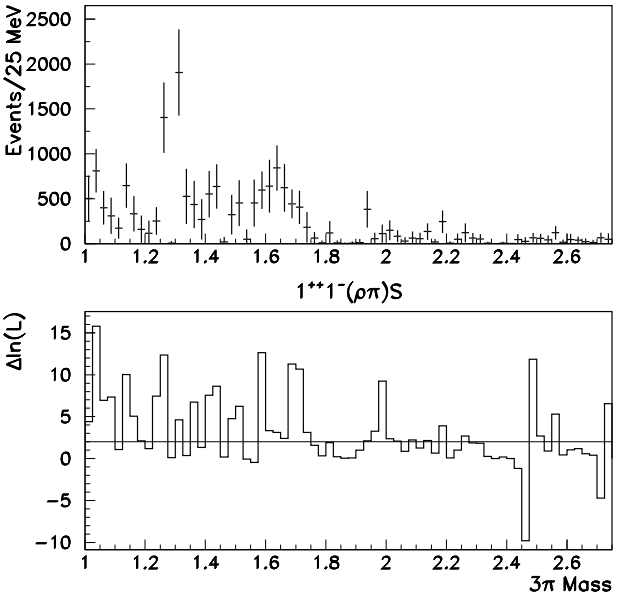

Figure 20: An insignificant partial wave

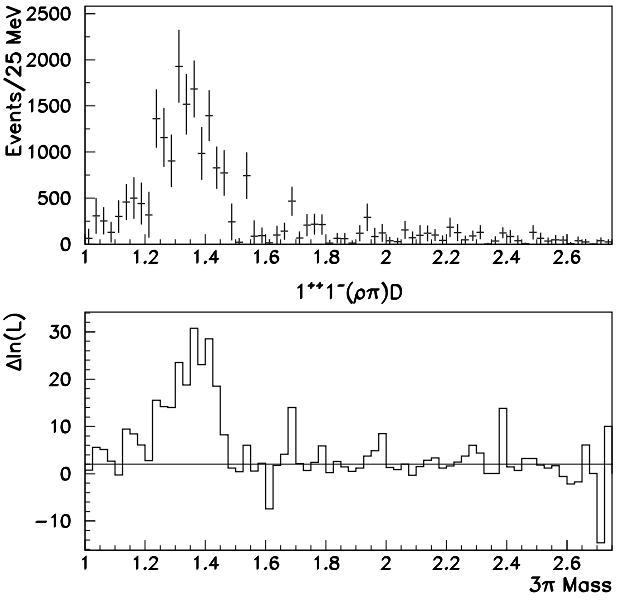

Figure 21: An insignificant partial wave

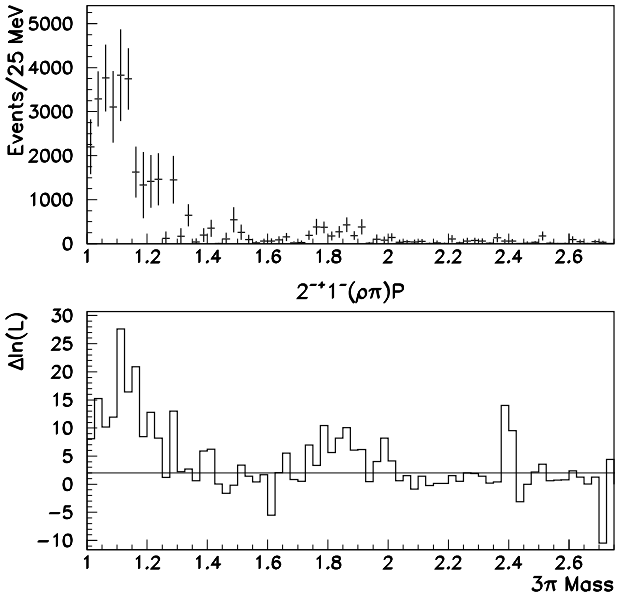

Figure 22: An insignificant partial wave

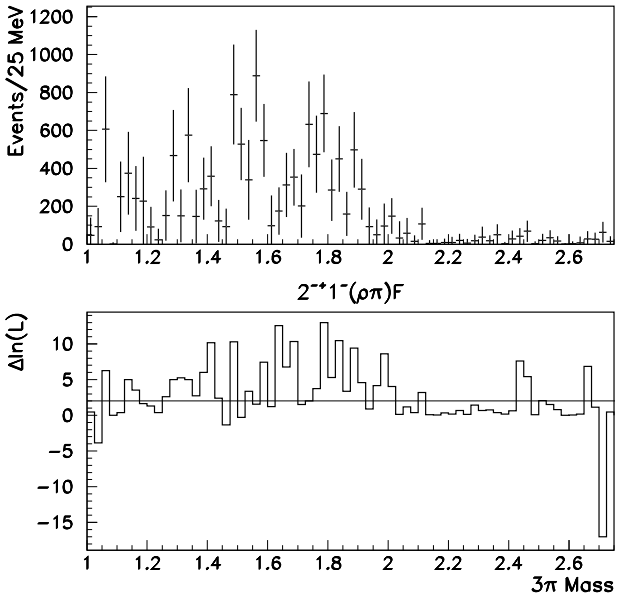

Figure 23: An insignificant partial wave

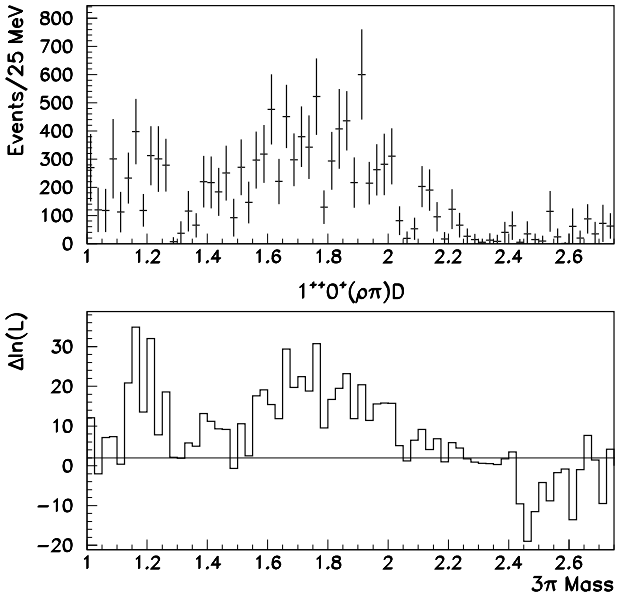

Figure 24: An insignificant partial wave

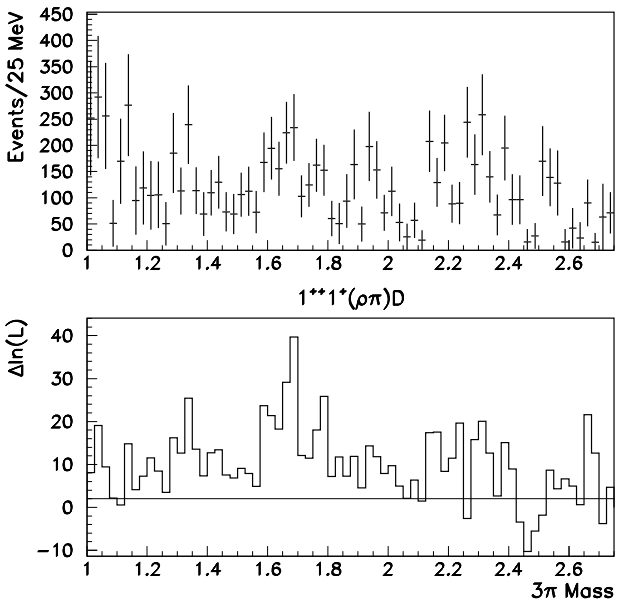

Figure 25: An insignificant partial wave

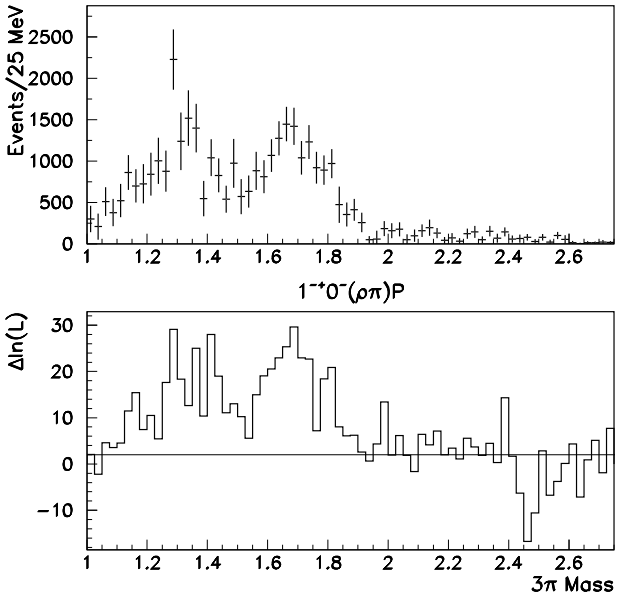

Figure 26: An insignificant partial wave that was kept. The quantum numbers are exotic and the existence/nonexistence of resonances in this partial wave is of interest.

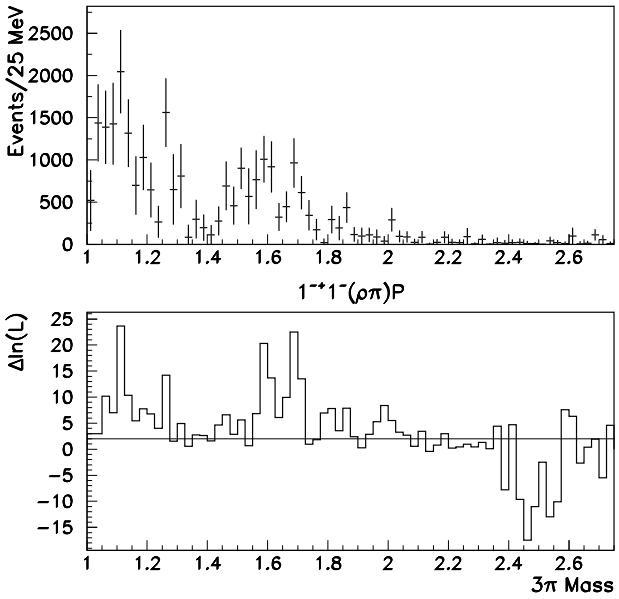

Figure 27: An insignificant partial wave that was kept. The quantum numbers are exotic and the existence/nonexistence of resonances in this partial wave is of interest.

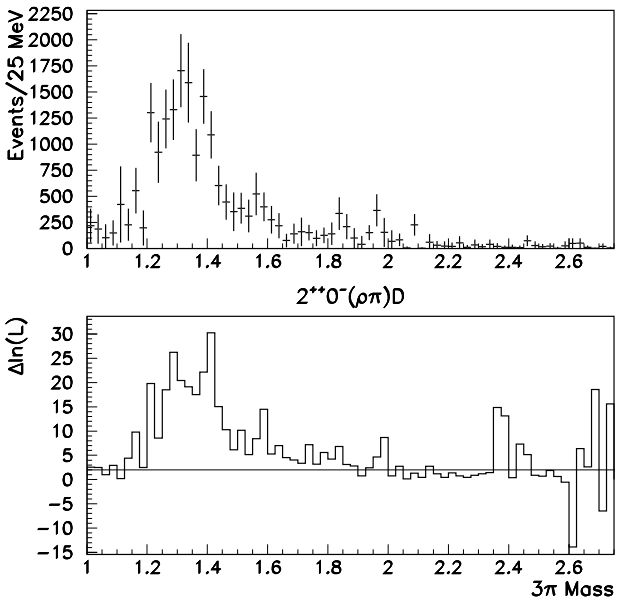

Figure 28: A marginally significant partial wave that was kept. $a_2(1320)$ production in negative reflectivity has been previously reported and the structure observed here seems consistent with that interpretation.

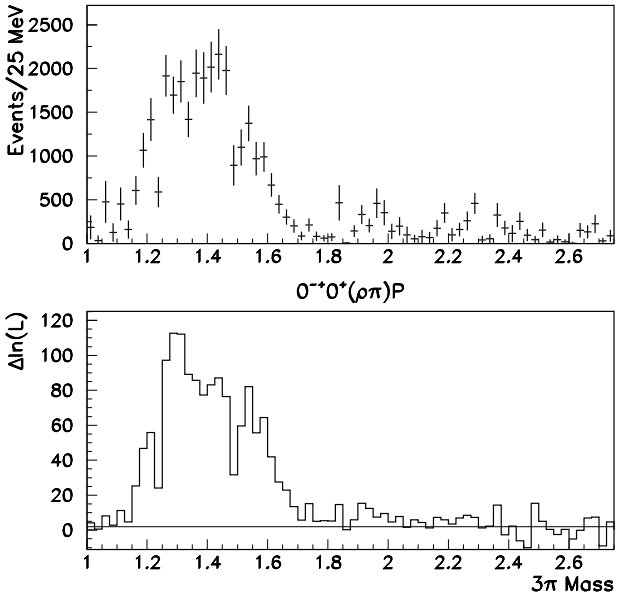

Figure 29: A significant partial wave

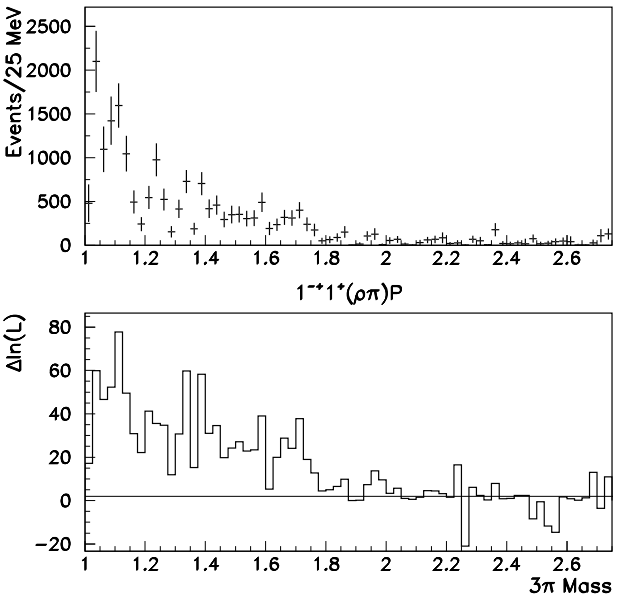

Figure 30: A significant partial wave

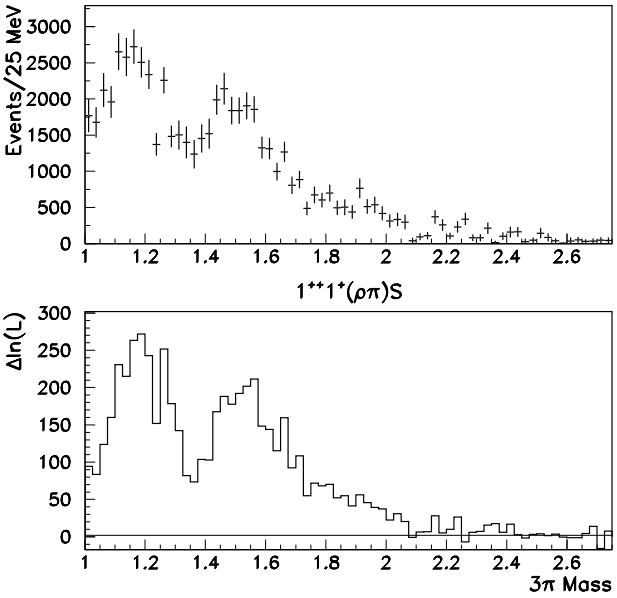

Figure 31: A significant partial wave

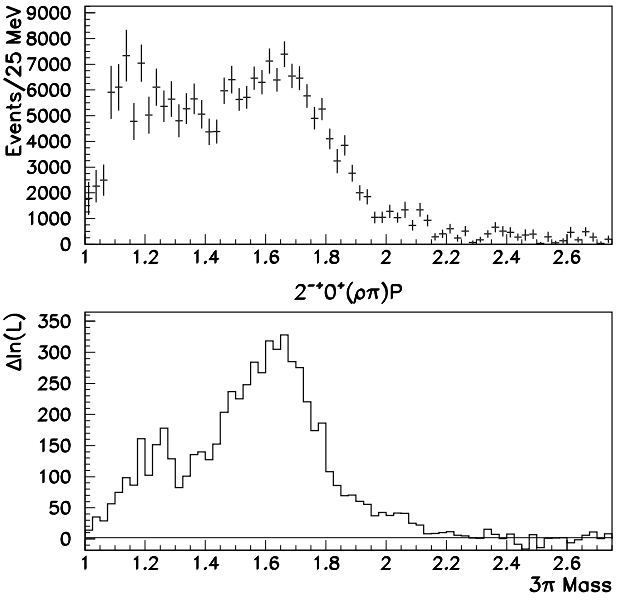

Figure 32: A significant partial wave

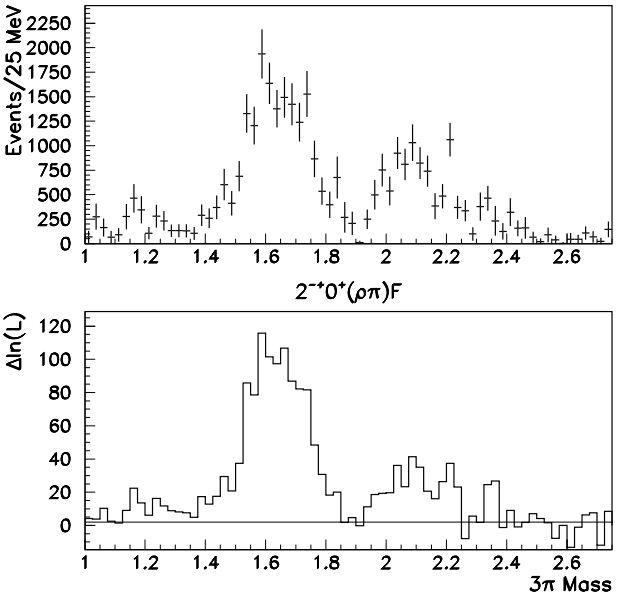

Figure 33: A significant partial wave

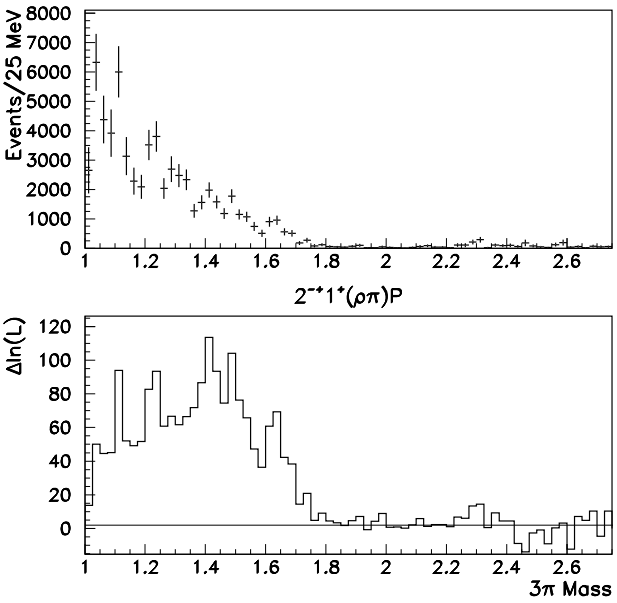

Figure 34: A significant partial wave

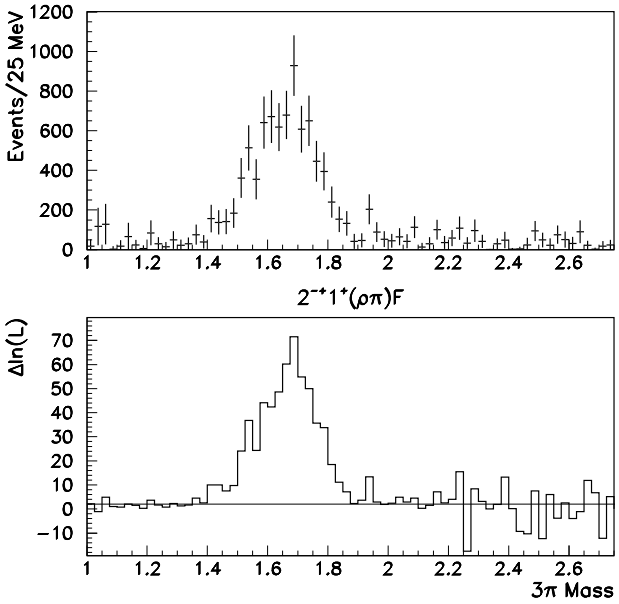

Figure 35: A significant partial wave

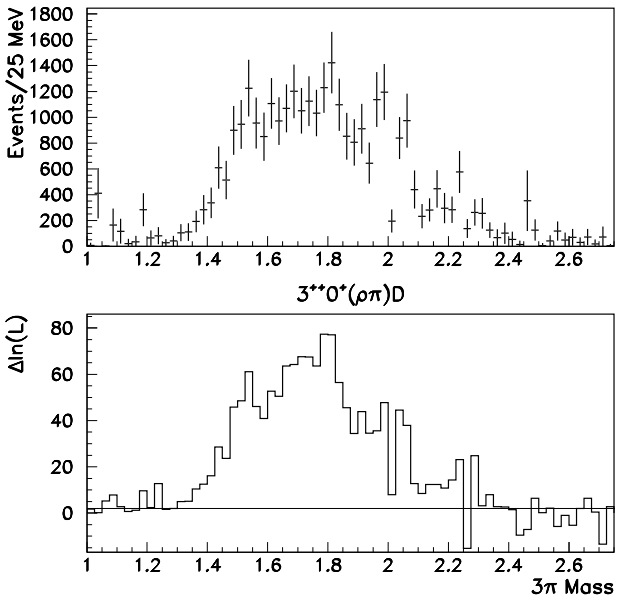

Figure 36: A significant partial wave

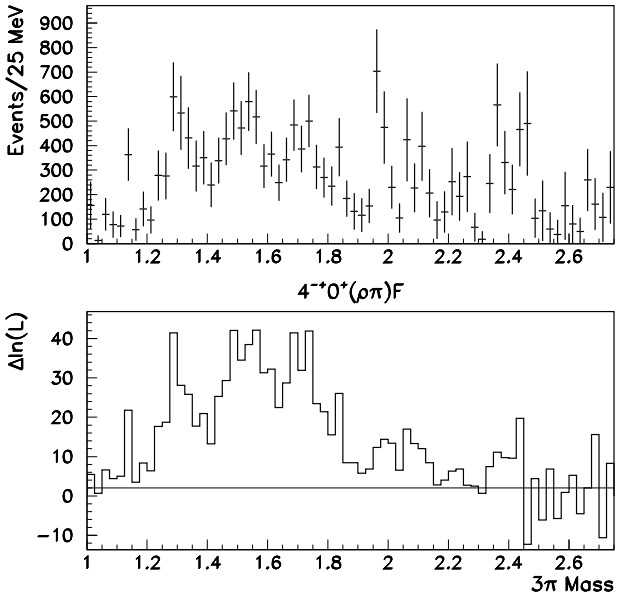

Figure 37: A marginally significant partial wave that was kept.

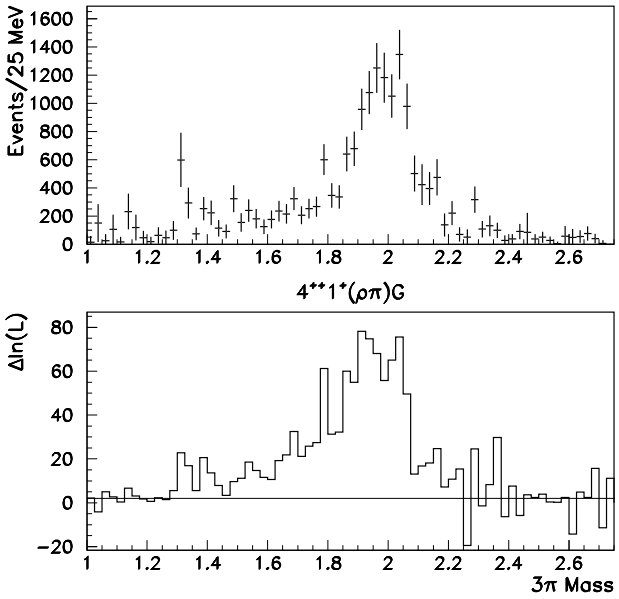

Figure 38: A significant partial wave