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Stability of the PWA Against Variations in the Selection Criteria *

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Abstract

The selection criteria defining the data sample used for Partial Wave Analysis (PWA) are varied and the effect on the PWA results are examined

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1 Introduction

To perform a PWA, a suitable data set must be produced. This involves selection of events from a parent data sample using various selection criteria, referred to as cuts. The purpose of these cuts is to remove events not of the type being considered ("noise") while preserving as many of the type desired as possible. The cuts chosen and their effect on the data is presented in detail in Note001 by R. Mitchell.

Often, the cuts effect the variables that determine the results of a PWA. The Monte Carlo, in principle should remove these artificial effects on the results. This study seeks to determine to what extent this is possible and to what extent the cuts effect that nature of the PWA results.

If a PWA result is to be believable, it should not depend strongly on the values of parameters that define the cuts. There may be some residual dependance of the results on the values of the cuts used and these differences can be used in estimating systematic errors. Large or structural differences in results under variation of the cuts indicates the analysis is unstable and should not be believed. It is shown here that under reasonable variation of cut parameters a particular PWA exhibits this required stability.

Other information is available from the techniques presented here. If it is known that a particular cut is essential for removing noise from the data, relaxation or removal of that cut and comparison with the original PWA can show the effect of the noise on the data if that noise were to contaminate the selected data sample. Estimates of the contamination and observation of the effects of the noise can again be used to estimate systematic errors.

Table 1 gives a list of the cuts used for the baseline data set used for PWA. Additional columns in that table give the perturbed values of the cuts studied in this note. It will be shown below that some of the cuts used in this analysis have no statistically significant effect on the analysis while others should be used in estimates of systematic errors on particle parameters determined by a PWA.

In the following sections, the methods used to compare two PWA are given and the effects of the cuts are discussed.

2 The Δ Test

To compare two PWA results the following quantity was computed for each partial wave and in each mass bin:

$$\Delta(M) = \frac{2 \mid A(M) - B(M) \mid}{\sqrt{\sigma_A^2 + \sigma_B^2}} \tag{1}$$

where A and B are intensities derived from each PWA. $\Delta(M)$ is interpreted as the distance between PWA A and PWA B at a mass M, calculated in units where the error of their average is one. Figure 1 shows a comparison of two PWA using this test. The top panel in this figure shows the intensities

Cut	Nominal	1c	1d1	1d2	1d3	1d4	1d5	1d6
Azimuth	$\pm 20°$							
Vertex_z	177 $\pm$14cm							
$\text{Vertex}_\text{perp}$	2.5σ							
$M_{p\pi^+}$	< 1.5 GeV	None						
CsI	20 MeV		None					
DEA veto	y			None				
Beam Hole	2.5σ				1.5σ	3.5σ		
Conf. Lev.	20%						10%	30 %

Table 1: Column 1 gives a list of cuts used (and described in the text) to select the baseline data set for PWA. Column 2 gives the value used for the cut in the baseline analysis. The additional columns give the internal names for the perturbed data sets and the values of the cuts used in each perturbed data set.

Cut	Nominal	1d8	1d9	1d10	1d11	1d12
Azimuth	$\pm 20°$	None				
Vertex_z	177 $\pm$14		177 $\pm$20cm	177 $\pm$8		
$\text{Vertex}_\text{perp}$	$2.5\ \sigma$				1.5σ	$3.5\ \sigma$

Table 2: Table 1, continued

obtained from two PWA superimposed on each other. The bottom panel shows $\Delta(M)$ for these two intensities. Clearly, values of Δ large compared to one indicate disagreement. No systematic disagreement between these intensities is noted although there are two anomalous mass bins.

3 The Δ' Test

Suppose two analyses differ everywhere by a multiplicative factor. This can occur, for example, by removing events at random from one data set and not from the other. If this is the case the Δ test can be modified to use this single, multiplicative factor. Consider

$$\chi^2 = \sum_{ij} \left(\frac{(A_{ij} - kB_{ij})^2}{\sigma_{Ai}^2 + \sigma_{Bij}^2} \right) \tag{2}$$

where i indexes mass bins and j indexes partial waves. Setting

$$\frac{d\chi^2}{dk} = 0 \tag{3}$$

determines k. Note that k is a single multiplicative factor applied to *all* partial wave intensities in one of the PWA being compared.

If this factor is applied universally to the comparison of two analyses a new test

$$\Delta'(M) = \frac{2 \mid A(M) - kB(M) \mid}{\sqrt{\sigma_A^2 + (k\sigma_B)^2}} \tag{4}$$

is obtained.

To summarize, two PWA that are statistically identical will have $\Delta(M)$ small compared to one for all mass and all partial waves. Two PWA that differ only by a multiplicative factor will have $\Delta'(M)$ small everywhere.

The effects of the cuts are discussed sequentially in the following sections.

4 Confidence Level

The data selection code returned a value of the confidence level that the event satisfied the hypothesis $\pi^- p \to \pi^+\pi^-\pi^- p$ and conservation of energy/momentum. For a data sample consisting entirely of events of this type, no contamination and perfectly understood resolution the distribution of confidence level values is uniform between 0 and 1.

The nominal value of the Confidence Level (CL) cut is 20%. In addition to the PWA at the nominal value, two PWA using CL cuts of 10 and 30% were performed. Lowering the value of the CL cut admits non-3π events, raising it removes signal events. If the nominal value of the cut is sufficient to

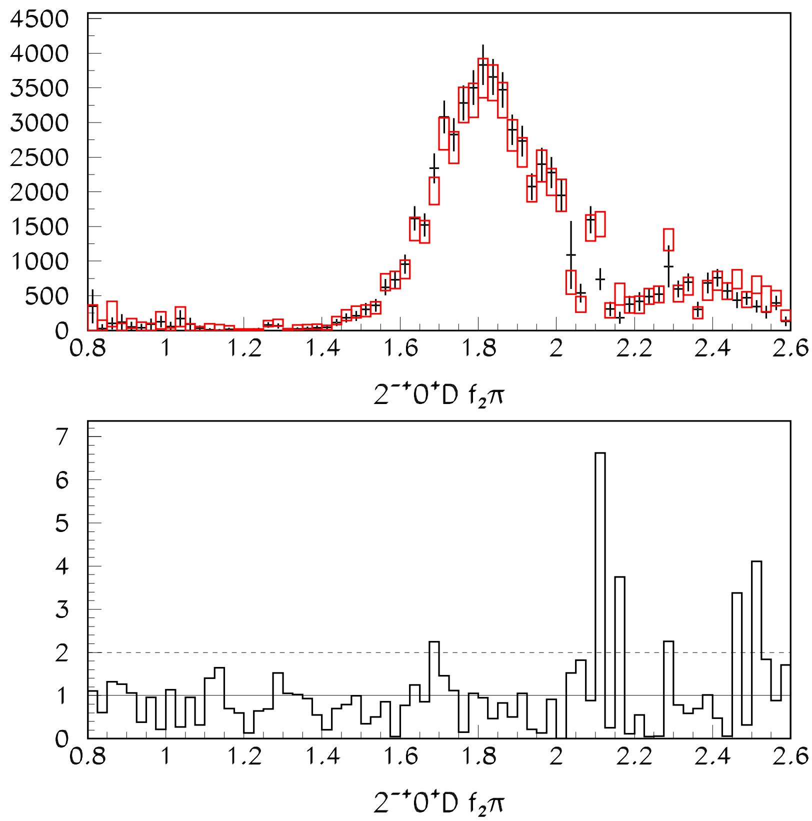

Figure 1: Top: the intensity of a partial wave from two different PWA. Bottom: The distance between the two results normalized to the error of their average, as a function of mass. Disagreement is observed in in a few bins where the fits were exhibiting instability.

remove the noise from the signal, using values larger that the nominal simply reduce the statistics of the sample without improvement in the signal to noise ratio.

If a more severe CL cut does not change the results of the PWA, it can be concluded that the contributions due to noise have been adequately suppressed by the cut using its nominal value. Analysis of the CL=30% data sample showed no significant difference between this sample and the sample using the nominal value. The Δ test, described above, identified the $1^{++}\rho\pi$ † partial wave to be most sensitive to an increase in the value of this cut. Figure 2 shows the two results. A fit to the two peaks using a gaussian gave indistinguishable values for the mean and width of the distributions. That is to say, while the overall number of events associated with the $a_1(1260)$ changed by an estimated 6%, the mass and width parameters were unchanged. The Δ' test (see figure 3) showed these PWA differ only by a numerical scale factor.

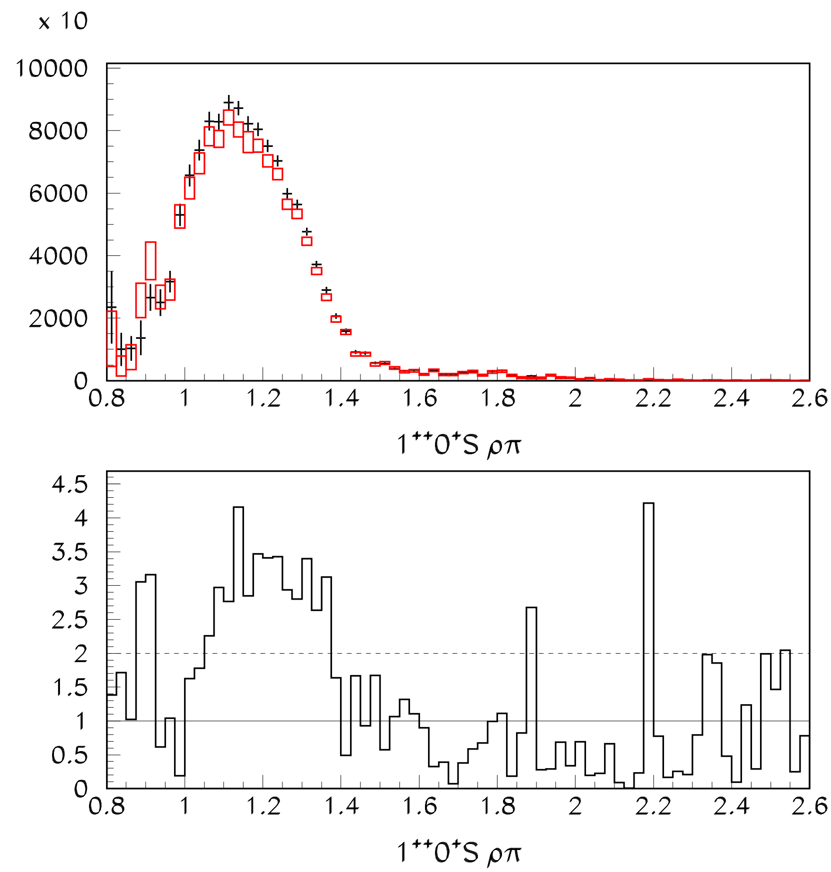

Figure 2: The intensity of the $1^{++}\rho\pi$ partial wave using the nominal value of the CL cut (20%, points with error bars) and a more severe value. (30%, boxes) and (bottom) $\Delta(M)$

A less severe value of the CL cut admits noise and analysis of a data sample using a smaller value for the cut demonstrates the effect of noise on the data. Analysis of the CL=10% data sample showed no significant difference between this sample and the sample using the nominal value. The $1^{++}\rho\pi$ partial wave was again identified by the Δ test as most effected. Figure 4 shows the effect.

† This is the dominant decay channel of the $a_1(1260)$ and the largest single partial wave in the data at low t

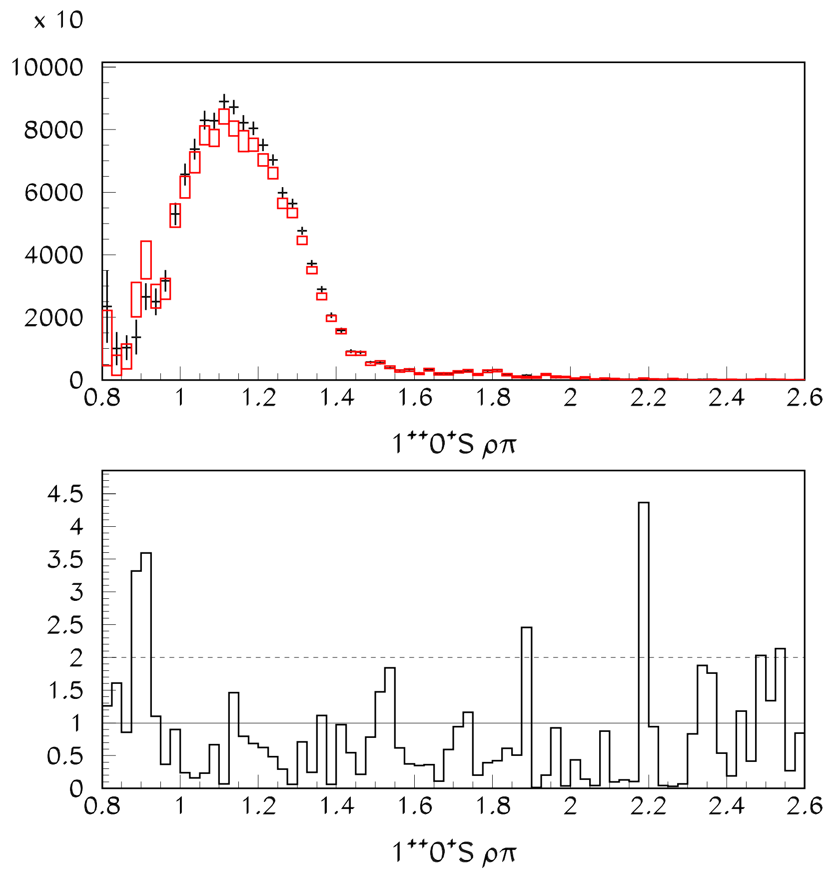

Figure 3: Top, as in figure 2, bottom, $\Delta'(M)$ showing the results differ only by a multiplicative factor.

It can be seen, reducing the CL cut value increases the number of events. The Δ' test shows that these two PWA, again, differ only by a multiplicative factor.

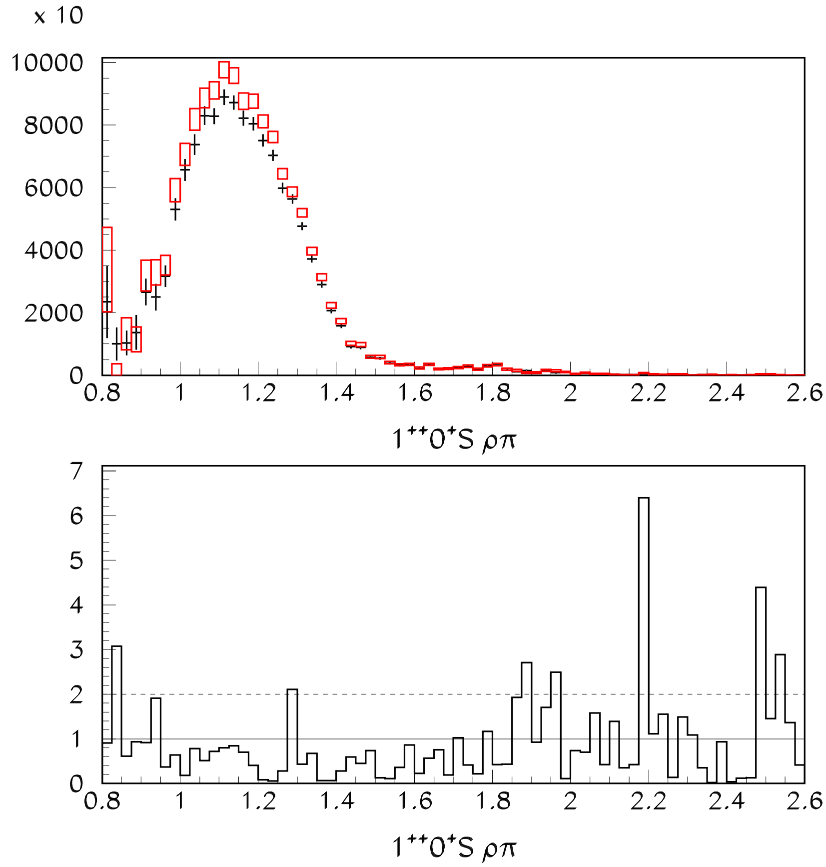

Figure 4: The intensity of the $1^{++}\rho\pi$ partial wave using the nominal value of the CL cut (10%, points with error bars) and a less severe value and (bottom) $\Delta'(M)$

It is concluded that, apart from multiplicative factors, the value of the CL cut does not effect the results of the PWA. The shape of the confidence level distribution indicates that a value of 20% is reasonable.

5 The Cesium Iodide Cut

The target was surrounded by an array of Cesium Iodide (CsI) crystals instrumented with photodiodes to reject events with a low energy π^0. Low energy pions are produced, for example, by

$$\pi^- p \to X^- \Delta^+ \qquad X^- \to \pi^+\pi^-\pi^- \qquad \Delta^+ \to p\pi^0. \tag{5}$$

In this case, the π^0 from the Δ decay can be produced at large angles to the beam and at low energies. The CsI array was specifically intended to identify and reject these types of events.

To examine the effect of selecting events with no signal in the CsI detector PWA with and without this selection were performed. While the data sample without this selection is 1.66 times larger than the sample without, the Δ' test identified *no significant differences* between these analyses. Figures 5 and 6 show two large intensity partial waves and the result of the Δ' test.

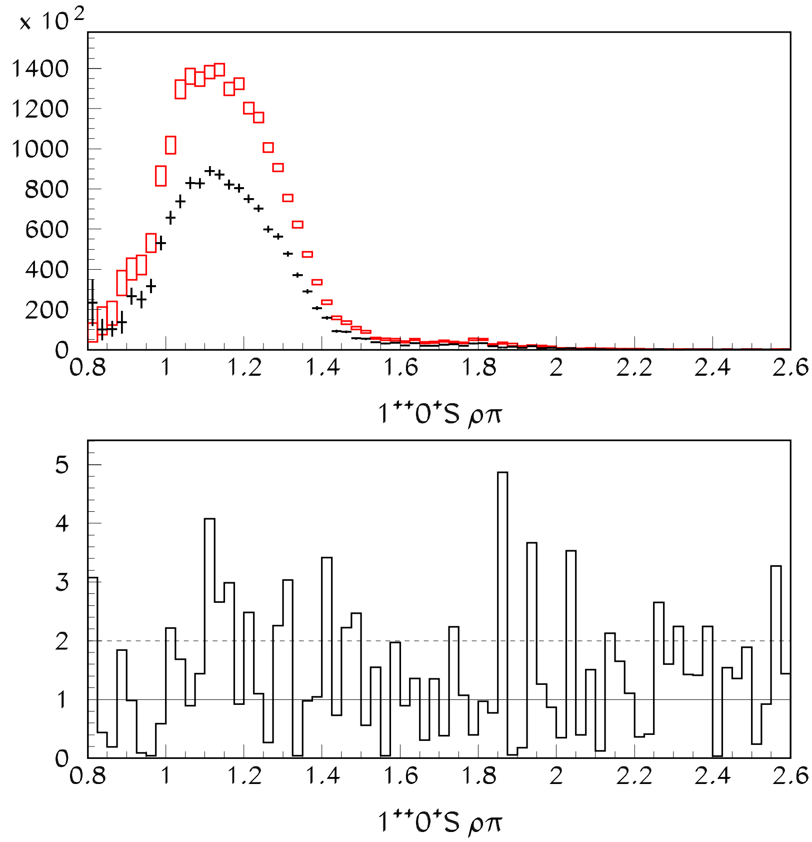

Figure 5: A comparison of the result of two PWA with and without the CsI cut described in the text. The intensities differ by a significant factor but the shapes of intensities are very similar. The bottom panel shows the result of the Δ' test, a test insensitive to differences of scale.

6 The Δ^{++} cut

Consider the following reaction and subsequent decay:

$$\pi^- p \to \pi^- \pi^- \Delta^{++} \qquad \Delta^{++} \to p\pi^+. \tag{6}$$

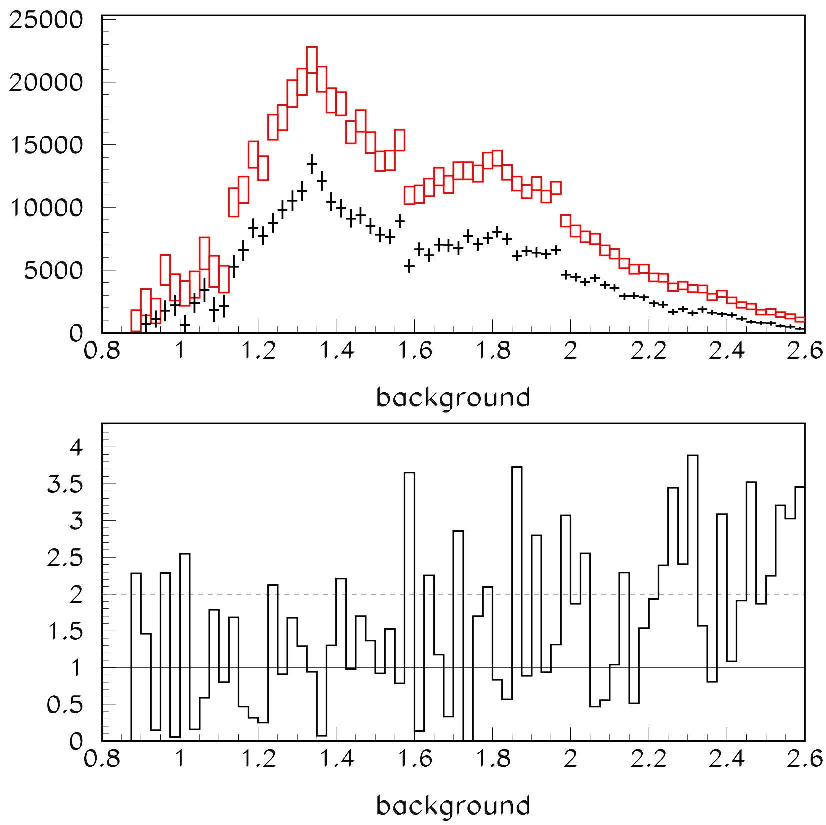

Figure 6: The effect of the CsI cut, described in the text, on the background.

The final state is identical to that being considered but the pions are not produced via decay of a single, negatively charged meson. To eliminate events from this source, it was required that the $p\pi^+$ effective mass be less than 1.5 GeV. Two PWA were performed, with and without this requirement. It was found that these PWA were different but these differences observable only at large 3π effective mass and only where the intensities under consideration were small. Put another way, removing this cut effected the high mass tail of small intensities and had no effect on large, resonant signals. Figure 7 shows the effect of this cut on the large $a_2(1320) \to \rho\pi$ signal and figure 8 the effect on the large $\pi_2(1670) \to f_2\pi$ signal.

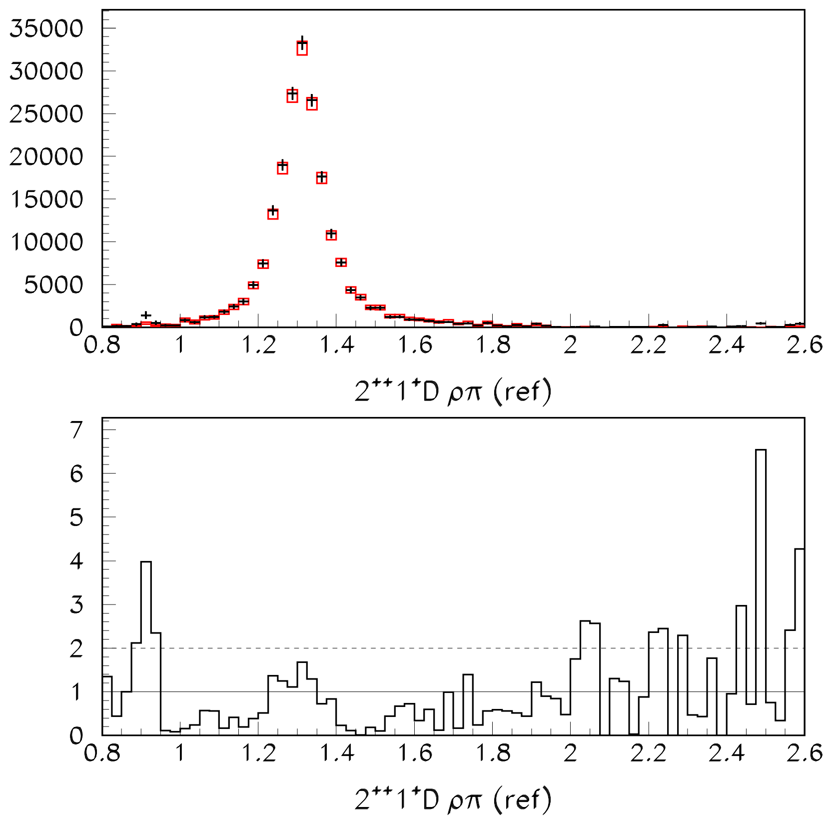

Figure 7: The effect of the Δ^{++} cut on a large, resonant partial wave. Disagreement in the "tail" is apparent.

7 Beam Hole Cut

During the 1995 run of E852 the beam rate was high enough to introduce inefficiency in the trigger PWCs near the beam path. The efficiency was not zero, nor was its spatial extent sharp-edged. These conditions are difficult to simulate so a region around the beam at the PWC planes used in the trigger was defined. Events with tracks projected into these elliptical regions were rejected. This rejection is well defined and sharp-edged and therefore easier to simulate.

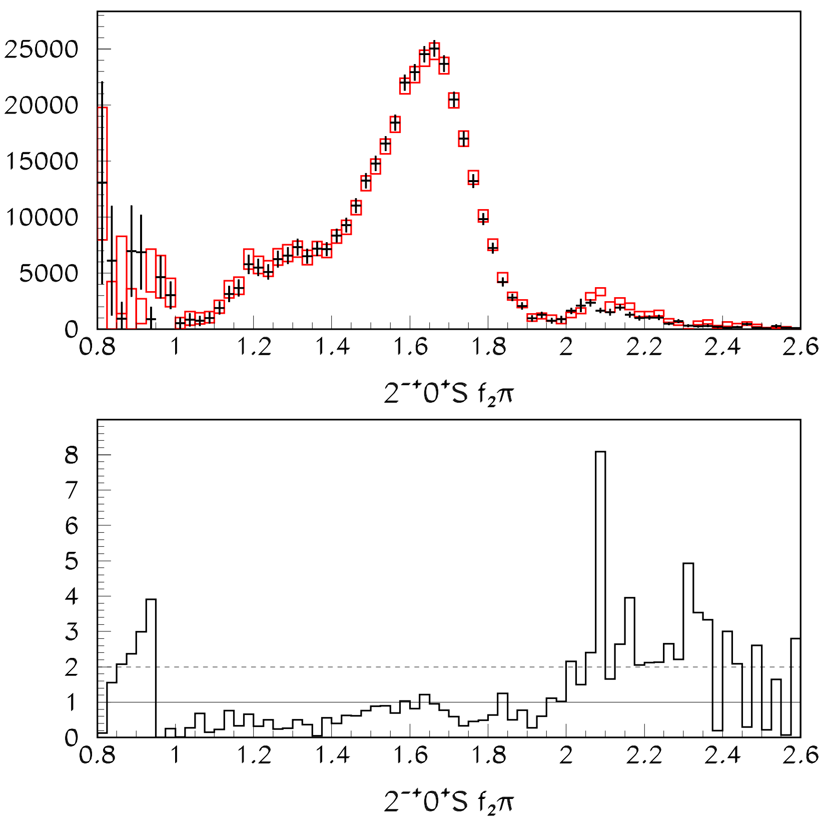

Figure 8: The effect of the Δ^{++} cut on a large, resonant partial wave. Disagreement in the "tail" is apparent.

The cut was implemented by defining a distance

$$R = \sqrt{\left(\frac{X - X_{beam}}{\sigma_X}\right)^2 + \left(\frac{Y - Y_{beam}}{\sigma_Y}\right)^2} \qquad (7)$$

where $X_{beam}, Y_{beam}, \sigma_X$ and σ_Y are the mean position and width of the beam at the appropriate trigger PWC. If (X, Y) is the position of a particle when $Z = Z_{PWC}$, R can be interpreted as the distance, in units of σ, of that particle from the average beam position. Tracks with a small value of R traversed the inefficient region of the trigger PWCs and were eliminated with the Beam Hole (BH) cut.

The nominal value of the beam hole cut was $R < 2.5$. Analyses were performed with values of 1.5 and 3.5 to investigate the effect of this cut.

This cut was the first examined that produced a difference in the results that could not be explained by a simple change of scale applied to all intensities. Figures 9 and 10 show a comparison between the PWA using the nominal value of this cut and a less (more) severe value. In these figures, the result from the perturbed PWA is shown scaled by the factor k as in eqn. 3. Any parameters determined for the $a_1(1260)$ must include a systematic error associated with the variation of this cut.

In addition to the differences observed in the large, resonant $1^{++}\rho\pi$ partial wave, a difference in the non-resonant $0^{-+}\rho\pi$ partial wave (see figs. 11 and 12) was observed. The less severe cut showed an anomalous reduction in the number of events at low mass. The more severe cut showed non-monotonic behavior at low mass. This non-monotonic behavior in this region of mass must be viewed with suspicion and conclusions based on this partial wave made with caution.

8 The DEA cut

There was a "picture frame" photon veto counter that was sensitive to low energy photons that are outside the geometric acceptance of the Lead Glass Detector. The counter was referred to as the "Downstream Endcap Array" or DEA. Covering the DEA were scintillators sensitive to charged particles. An event with soft photons was vetoed (off line) if a signal existed in DEA but not in a corresponding scintillator element. To investigate the effect of this veto, two PWA, with and without this software veto, were performed.

This selection, again, introduces a systematic bias in the results. Figures 13 and 14 show two targe, resonant partial waves with and without the DEA cut. Clearly, any particle parameters deduced from these distributions require the addition of a systematic error associated with this selection.

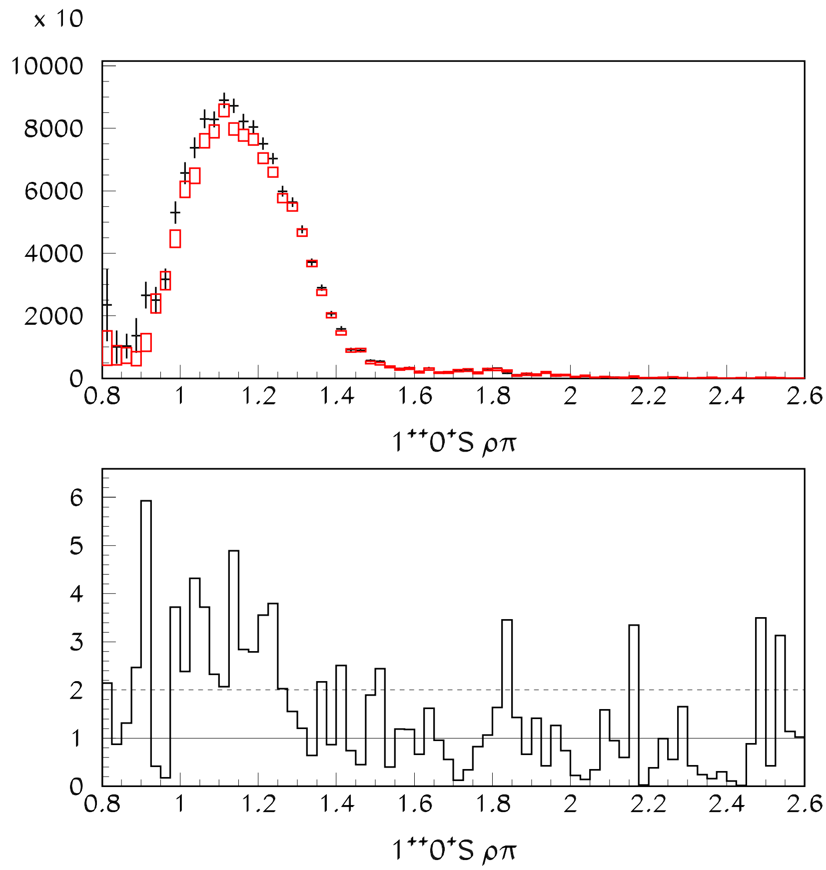

Figure 9: The intensity of the $1^{++}\rho\pi$ partial wave using the nominal value of the Beam Hole cut (2.5, points with error bars) and a less severe value (1.5, boxes) and (bottom) $\Delta'(M)$. The scaling factor from eqn. 3 has been applied to the result shown as boxes. Disagreement is seen across the entire $a_1(1260)$ region.

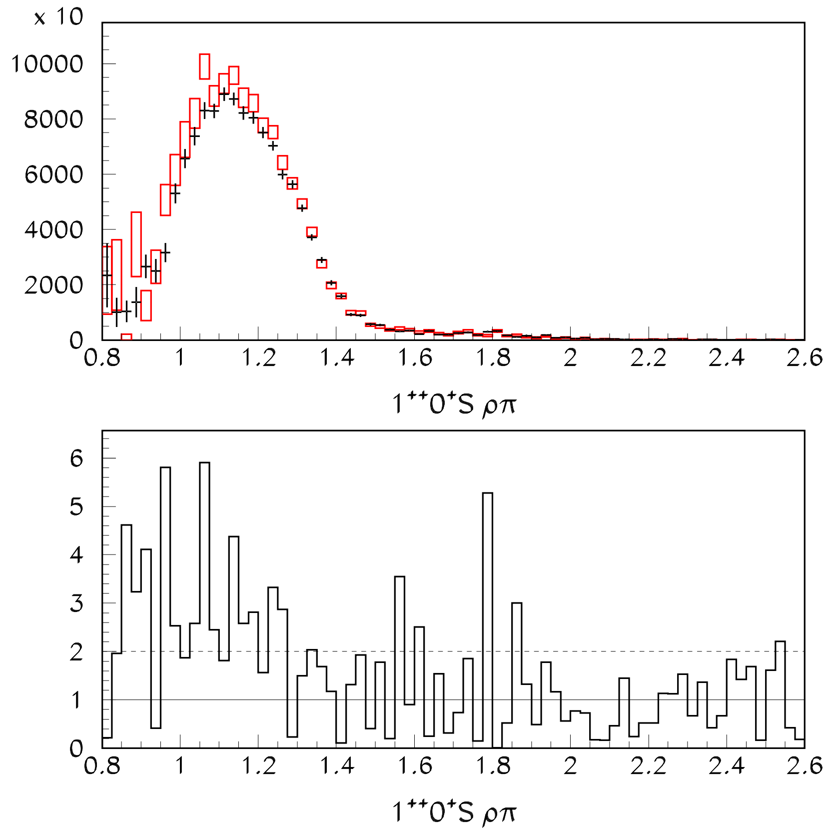

Figure 10: The intensity of the $1^{++}\rho\pi$ partial wave using the nominal value of the Beam Hole cut (2.5, points with error bars) and a more severe value (3.5, boxes) and (bottom) $\Delta'(M)$. The scaling factor from eqn. 3 has been applied to the result shown as boxes. Disagreement is seen across nearly the entire $a_1(1260)$ region.

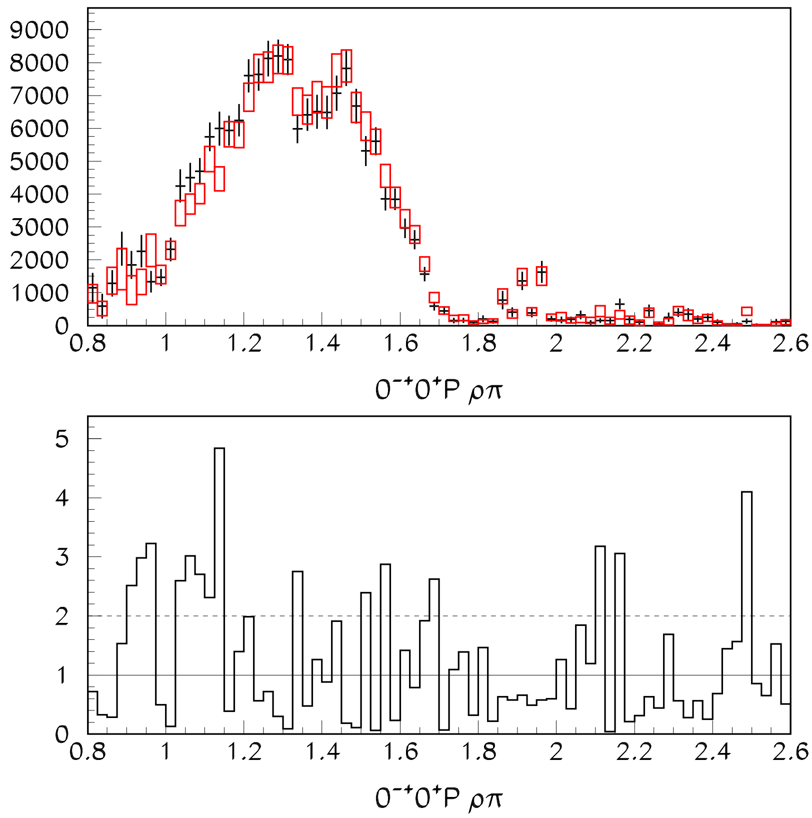

Figure 11: The intensity of the $0^{-+}\rho\pi$ (non-resonant) partial wave using the nominal value of the Beam Hole cut (2.5, points with error bars) and a less severe value (1.5,boxes) and (bottom) $\Delta'(M)$. Disagreement is seen below 1.2 GeV.

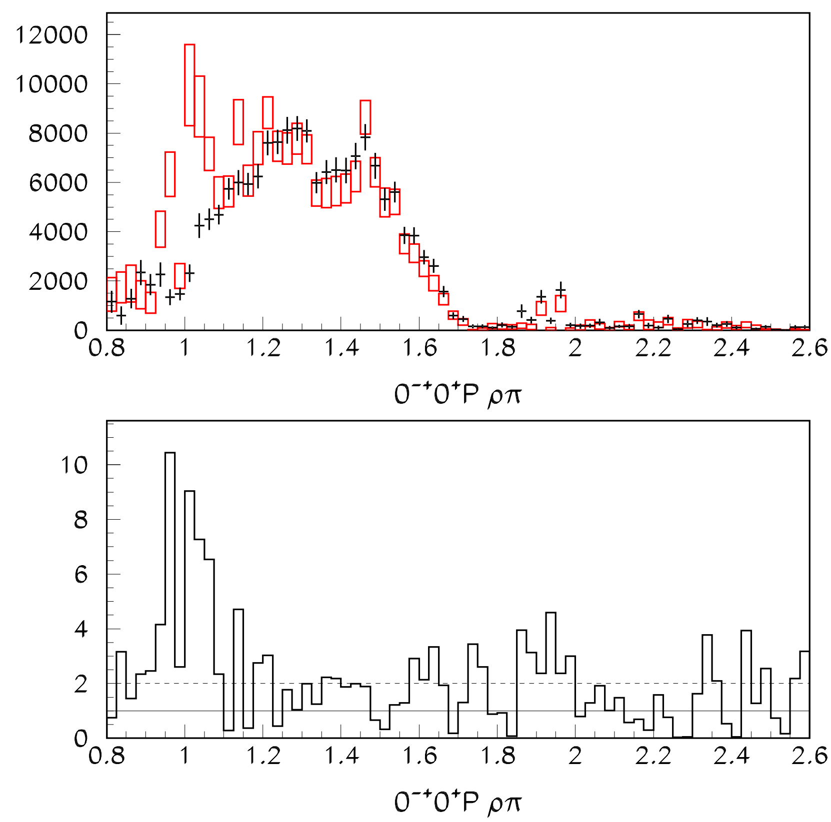

Figure 12: The intensity of the $0^{-+}\rho\pi$ (non-resonant) partial wave using the nominal value of the BH cut (2.5, points with error bars) and a more severe value (3.5, boxes) and (bottom) $\Delta'(M)$. Instability in the perturbed analysis is seen below 1.2 GeV.

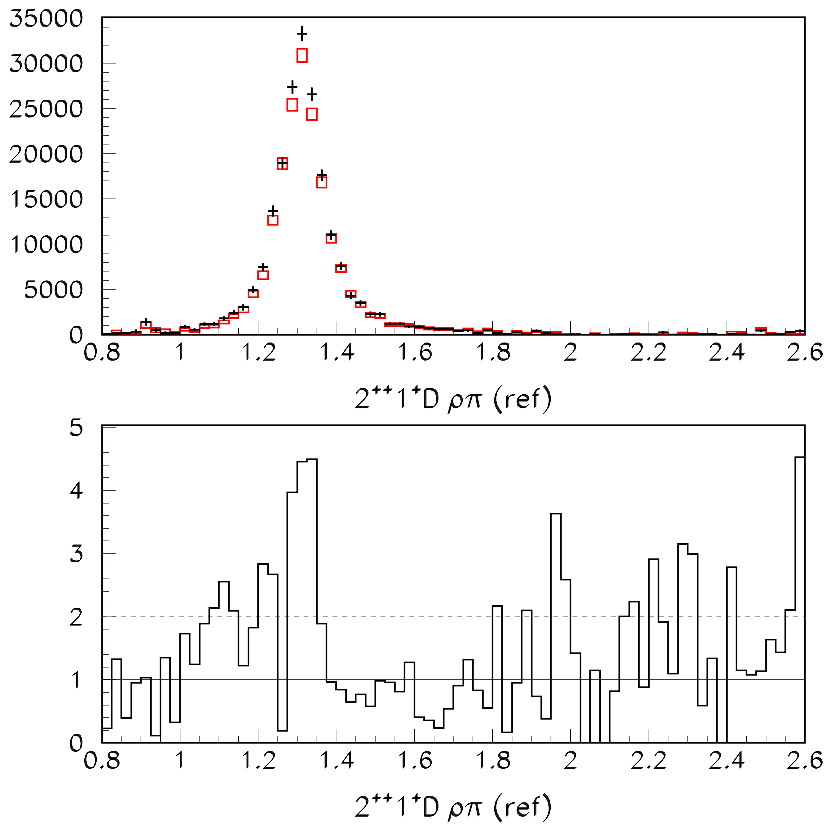

Figure 13: The $2^{++}\rho\pi$ partial wave with (points) and without (boxes) the DEA cut described in the text. The Δ' test (bottom) shows this cut effects the low edge of the $a_2(1320)$ differently than the high edge.

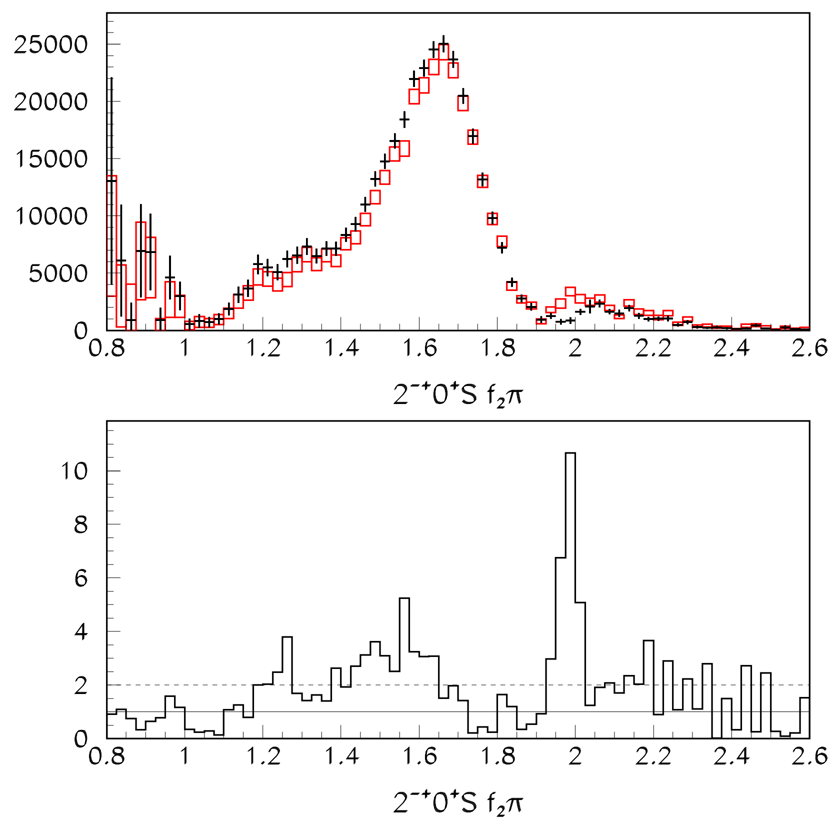

Figure 14: The $2^{++}\, f_2(1275)\pi$ partial wave with (points) and without (boxes) the DEA cut described in the text. The Δ' test (bottom) shows this cut effects the low edge of the $\pi_2(1670)$ differently than the high edge.