IU Task D Note-009
Issued: February 9, 2005

Calculation of Moments *

Adam Szczepaniak, Maciej Swat and Scott Teige
Department of Physics
Indiana University, Bloomington, IN 47405

Abstract

The angles used to calculate experimental moments are defined.

*Work supported by Department of Energy contract DE-FG02-91ER40661.

All angles used in PWA are evaluated in the center of momentum of the 3π system. The Gottfried-Jackson (GJ) frame is defined to be the center of momentum frame rotated such that the z axis is parallel to the beam and the y axis is along the normal to the production plane.

Considering the case where 3π is $\pi^-\pi^-\pi^+$ let the momentum in the GJ frame of one π^- be given by

$$\vec{p} = p\hat{u} = p \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \tag{1}$$

If

$$R(\theta,\phi) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & -\cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

then

$$R\vec{u} = -\hat{z}. \tag{3}$$

That is, the rotation R takes the π^- momentum vector into the minus z-axis.

Defining

$$\begin{aligned} \beta &= \pi - \theta \\ \gamma &= \phi - \pi, \end{aligned} \tag{4}$$

$$R(\beta,\gamma) = \begin{bmatrix} -\cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} -\cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & -\cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

Let the momentum of the π^+ (in the GJ frame) be denoted as $\vec{q}$ and

$$\vec{q}\,' = R\vec{q} \tag{6}$$

Writing

$$\vec{q}\,' = \mid \vec{q} \mid \begin{bmatrix} \sin\zeta\cos\alpha \\ \sin\zeta\sin\alpha \\ \cos\zeta \end{bmatrix} \tag{7}$$

defines α.

The rotation

$$\vec{q}\,'' = Q\vec{q}\,' \tag{8}$$

with

$$Q = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

takes $\vec{q}\,'$ into the $x - z$ plane. The transformation T from the GJ frame to the "standard configuration" is then

$$T = Q(\alpha)R(\beta,\gamma). \tag{10}$$

Figure 1 shows an event in the standard configuration.

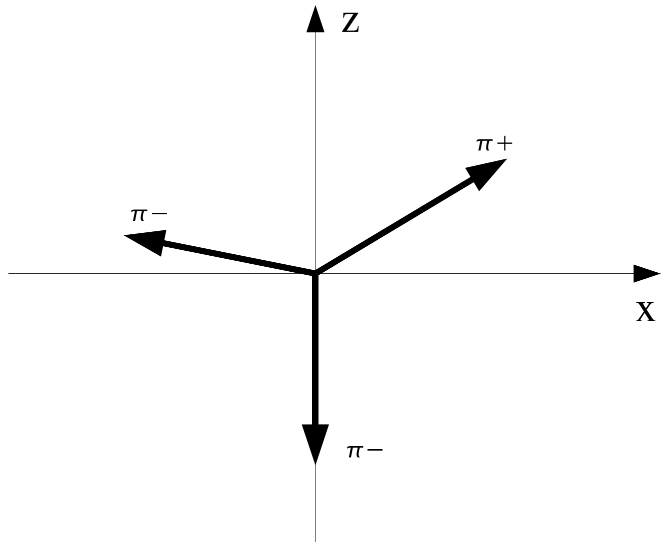

Figure 1: Standard configuration

The initial choice of π^- was arbitrary. Choosing the other π^- gives three other, equally physically significant, angles $(\alpha', \beta', \gamma')$ The angles for the $\pi^0\pi^0\pi^-$ case can be calculated with the replacement $\pi^- \to \pi^0$ and $\pi^+ \to \pi^-$ in the above.

The moments are then defined to be

$$H_{LMN} = \frac{1}{2} \sum_{data} D^L_{MN}(\alpha, \beta, \gamma) + D^L_{MN}(\alpha', \beta', \gamma') \quad (11)$$

where D are the Wigner D-functions.